which divides the pole into upper and lower segments equal to $(l/2)(2 - \sqrt{2})$ and $(l/2)\sqrt{2}$ respectively, the pole then forming an isosceles triangle with the axes.

Excellent solutions were received from FRANK IRWIN, J. A. CAPARO, H. C. FEEMSTER, and N. P. PANDYA.

CALCULUS.

384. Proposed by JOSEPH B. REYNOLDS, Lehigh University.

In what time will a sum of money double itself at 6 per cent. interest per annum if compounded at indefinitely short intervals?

SOLUTION BY H. L. AGARD, Williams College.

If the interest is compounded k times a year, the amount after n years is given by the formula

$$A = P\left(1 + \frac{r}{k}\right)^{nk}.$$

When the interest is compounded at indefinitely short intervals,

$$A = \lim_{k=\infty} P\left(1 + \frac{r}{k}\right)^{nk} = \lim_{k=\infty}\left[P\left(1 + \frac{r}{k}\right)^{k/r}\right]^{nr} = Pe^{nr}. \tag{1}$$

In (1), setting $A = 2P$, $r = .06$ and solving for n, we have

$$n = \frac{\log_e 2}{.06} = \frac{.69315}{.06} = 11.5525 \text{ years.}$$

Also solved by H. C. FEEMSTER, W. W. BURTON, G. W. HARTWELL, C. E. FLANAGAN, J. W. CLAWSON, FRANK R. MORRIS, H. S. UHLER, ELIZABETH OVIN, F. FORDERO (Seville, Spain), and and HERBERT N. CARLETON.

386. Proposed by HERBERT N. CARLETON, West Newbury, Mass.

C is a fixed point on the perpendicular bisector of a line segment AB. Locate a point D also on this bisector, such that $AD + BD + DC$ shall be a minimum.

SOLUTION BY H. C. FEEMSTER, York College, Nebraska.

Let the foot of the perpendicular be E, $CE = b$, $AE = EB = a$, and $DE = x$. Then $AD + BD + DC = 2\sqrt{a^2 + x^2} + b - x$, which is to be a minimum. Taking the derivative of this expression, setting it equal to 0, and solving for x, we have

$$x = \frac{a\sqrt{3}}{3} = DE, \text{ as required.}$$

MECHANICS.

301. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

A wire is hanging from two points in the same horizontal plane. If the difference between the length of the wire and the actual distance between the supports is very small, show that

$$s = x\left(1 + \frac{x^2}{6c^2}\right),$$

where s is one half of the length of the wire, c is the tension at the lowest point divided by w the load per unit of horizontal distance, and x is the distance of the lowest point of the curve to the point of support.

SOLUTION BY A. M. HARDING, University of Arkansas.

A solution of this problem will be found in Jean's *Theoretical Mechanics*, pages 80–85. It is there shown that the length of the wire is given by

$$s = \frac{c}{2}\left(e^{x/c} - e^{-x/c}\right).$$

If we expand $e^{x/c}$ and $e^{-x/c}$ as power series in x and neglect all powers of x higher than the third we obtain the desired result.

Similarly solved by WALTER C. EELLS and R. M. MATHEWS.

302. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

A ball is projected from a given point at a given inclination β towards a vertical wall; determine the velocity so that after striking the wall the ball may return to the point of projection.

SOLUTION BY PAUL CAPRON, Annapolis, Maryland.

Let v = the required velocity, a = the distance of the point of projection from the wall. After t seconds, the ball will have traversed x horizontally, y vertically, where

$$x = v\cos\beta\cdot t, \quad y = v\sin\beta\cdot t - \tfrac{1}{2}gt^2.$$

Its path will make the angle α with the horizontal, where $\tan\alpha = \tan\beta - (gt/v\cos\beta)$, and its component velocities will be $v\cos\beta$ horizontally, $v\sin\beta - gt$ vertically. Subscripting the values at the instant of impact, we have

$$x_1 = a, \quad t_1 = (a/v\cos\beta), \quad y_1 = a\tan\beta - (a^2g/2v^2)\sec^2\beta, \quad \tan\alpha_1 = \tan\beta - (ag/v^2)\sec^2\beta,$$
$$v_1^2 = v^2 - 2ag\tan\beta + (a^2g^2/v^2)\sec^2\beta = v^2 - ag\,(\tan\alpha_1 + \tan\beta).$$

The component velocities immediately after the impact will be

$$ev_1\cos\alpha_1 \text{ horizontally}, \quad ev_1(K\sin\alpha_1 - C\cos\alpha_1) \text{ vertically},$$

where, if e is the coefficient of restitution and μ is the coefficient of friction during the impact, $K = (5/7e)$ and $C = 0$ if $\mu > [2\tan\alpha_1/7(1+e)]$ or $K = 1/e$, $C = [\mu(1+e)/e]$ if $\mu < [2\tan\alpha_1/7(1+e)]$. [See Routh, *Elementary Rigid Dynamics*, Volume I, § 197. The ball is supposed homogeneous.]

Consequently, t seconds after the impact, the ball will have moved from the point of impact x horizontally and y vertically, where (supposing downward motion not to have begun) at the time of impact

$$x = ev_1\cos\alpha_1\cdot t, \quad y = ev_1(K\sin\alpha_1 - C\cos\alpha_1)t - \tfrac{1}{2}gt^2,$$

and it is required that when $x = a$, and therefore $t = (a/ev_1\cos\alpha_1)$,

$$y \text{ shall be } = -y_1 = -a\tan\beta + \frac{a^2g}{2v^2}\sec^2\beta,$$

i. e.,

$$a(K\tan\alpha_1 - C) - \frac{a^2g}{2e^2v_1^2}\sec^2\alpha + a\tan\beta - \frac{a^2g}{2v^2}\sec^2\beta = 0.$$

If we substitute in this equation the values of v_1^2 and $\tan\alpha_1$ given above, we shall have, after simplifying:

$$2e^2[(1+k)\tan\beta - C]v^6 - ag[(1+e^2+2Ke^2)\sec^2\beta + 4(1+K)e^2\tan^2\beta - 4e^2C\tan\beta]v^4$$
$$+ 2a^2g^2\sec^2\beta[(1+2e^2+3Ke^2)\tan\beta - e^2C]v^2 - a^3g^3(1+e^2+2Ke^2)\sec^4\beta = 0.$$

If we let

$$\frac{1+e^2+2Ke^2}{(1+K)\tan\beta - C} = 4e^2r \quad\text{and}\quad \frac{v^2}{ag} = x,$$

we shall have

$$x^3 - 2(r\sec^2\beta + \tan\beta)x^2 + \sec^2\beta(4r\tan\beta + 1)x - 2r\sec^4\beta \equiv$$
$$(x - 2r\sec^2\beta)(x^2 - 2\tan\beta x + \sec^2\beta) = 0.$$